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SEQUENTIAL BARGAINING WITH MANY BUYERS

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Number 366

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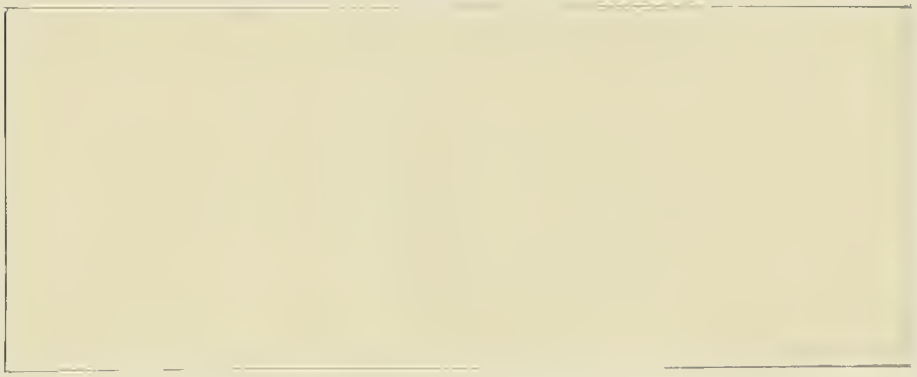

SEQUENTIAL BARGAINING WITH MANY BUYERS

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SEQUENTIAL BARGAINING WITH MANY BUYERS

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Abstract

We study bargaining between a single seller and a number of potential buyers of an indivisible object. The seller bargains with one buyer at a time, but may break off negotiations to bargain with someone else. We show that if switching buyers is costless the seller can credibly commit to a single take-it-or-leave it price, while if switching is costly the equilibrium may involve "haggling".

1. Introduction

A seller is the sole supplier of a single indivisible good for which there are many potential buyers. The seller bargains with one buyer at a time, and makes all the offers. At any time the seller may break off negotiations in order to bargain with someone else. We have in mind, for example, an artist facing a stream of potential purchasers of her paintings.

We show that if the seller can costlessly switch buyers she can credibly set a single take-it-or-leave-it price, and, moreover, that this price is quite generally the same one the seller would charge could she precommit herself to any desired pricing path. Our results show that the Riley-Zeckhauser (1983) no-haggling result does not depend on the ability to precommit or on equal search costs for the seller and the buyers.

The seller always prefers to face many buyers, while each buyer would prefer to be the only trading partner. More surprisingly, total surplus can decrease in moving from one to many buyers, even though the total potential surplus is increased. Thus, given that there are many potential buyers, total surplus can sometimes be increased by requiring the seller to negotiate with only one of them.

Our many-buyer model of bargaining permits the endogenous determination of the seller's "value of outside opportunities" which is treated as exogenous in two-trader models. For example, if the good is produced for sale, it is natural to take the seller's consumption value of the good to be zero. However, in a many-buyer model this does not imply that the seller's reservation value is zero as well. The many-trader model makes clear that the fact that it is common knowledge there are gains from trade does not guarantee that players will continue to negotiate. Unsuccessful negotiations make the seller pessimistic, and may induce her to switch to a new buyer offering

better prospects.

Further, we find that with many buyers the effects of shrinking the time period between offers to zero enables the seller to appropriate all the surplus, in contrast to the one-buyer model in which a shorter time period transferred the surplus to the buyers. With many buyers and a short time period the seller can easily price-discriminate by switching buyers.

Finally, we consider the case in which the seller faces an additional delay cost in switching buyers. Because of this cost, the seller must become sufficiently pessimistic before switching buyers, so that the take-it-or-leave-it strategy may not be credible. In this case the equilibrium may involve "haggling": the seller initially makes a high offer; if this offer is refused, the seller's equilibrium payoff drops, but she follows with a lower offer to some buyer before switching. We give an example of a haggling equilibrium which duplicates the "high seller" equilibrium which Fudenberg-Tirole (1983) derived in a model with one buyer and two periods. Thus we derive endogoneously the two-offer-per-buyer constraint which was exogoneous in Fudenberg-Tirole.

2. The Model

The seller has a single indivisible object for sale. The seller derives no utility from having the object in her inventory, and storage is costless. It is common knowledge that the seller's valuation is zero. There are an infinite number of ex-ante identical buyers. Each buyer's valuation, b, is known only to him. The buyer's valuations are independently and identically distributed on the interval $[\underline{b},\bar{b}]$, $\underline{b} > 0$, according to the cumulative distribution function $F(b)$, which is common knowledge. Additional assumptions on F are discussed below. The seller and all buyers have discount

factors δ_S, δ_B, respectively.

Time is indexed by periods $t = 1, \ldots$. At time one the seller begins to bargain with the first buyer by naming a price which the buyer may accept or reject. If the buyer rejects, the seller has the choice of making a second offer to the same buyer in period 2, or to break off negotiations with this buyer and begin negotiations with another. If the seller does choose to switch buyers, she must wait a lag of d ("delay") periods before making her next offer. If $d = 0$ the seller can make an offer to the second buyer in period 2, so that switching is costless. Thus, the seller makes all the offers, and the seller cannot negotiate with several buyers at once. Because the buyers' valuations are independent, we assume that the seller cannot recall a buyer once having passed on to a new one.

A perfect Bayesian equilibrium of this game is a (history-contingent) sequence of prices and switching decisions for the seller, of buyers' acceptances or refusals of the offers, and of (updated) beliefs about the buyers' valuations satisfying the usual consistency conditions. The actions at each stage must be optimal given the beliefs and the opponents' strategies, and the beliefs must be derived from the actions by Bayes rules. This solution concept is a weaker version of the sequential equilibrium of Kreps-Wilson (1982) which is not defined for games with a continuum of actions. For a more formal definition of perfect Bayesian equilibrium see Fudenberg-Tirole (1983).

A useful fact about equilibrium is:

Lemma 1 (Successive Skimming): In any perfect Bayesian equilibrium, at any time, the seller's posterior about the valuation of the current buyer is the seller's prior truncated at some value β; that is, $\frac{F(b)}{F(\beta)}$ for $b < \beta$, 1 for $b > \beta$.

Proof: Omitted, see Fudenberg-Levine-Tirole (1985). The lemma follows from the "incentive-compatibility" constraints and the fact that high-value buyers lose more by waiting than low-value ones do.

In this paper we will restrict the notion of equilibrium to that of Markov-perfect equilibrium. A Markov-perfect equilibrium is a perfect Bayesian equilibrium that satisfies the additional restriction that the strategies depend only on the payoff-relevant aspects of the history, which are called the "state" of the game. In the present case the state is defined as the beliefs of the seller about the valuation of the buyers and (in the case of the buyer) the seller's latest offer. As the valuations of the various buyers are independently distributed, and the seller cannot recall buyers with whom she has previously broken off negotiations, the only state variable we need to keep track of is the seller's beliefs about the buyer she is currently facing. Thus, for our purposes a Markov-perfect equilibrium is a perfect Bayesian equilibrium in which the seller's actions depend only on her beliefs about the valuation of the "current" buyer, and not on his name or the history of past prices (except as this determines the seller's beliefs). Note that the buyer's strategy then depends only on the current beliefs and price.

3. No Delay Costs

This section analyzes the case in which switching buyers is not costly, so that $d = 0$. We also assume that the cumulative distribution function of buyers, $F(b)$, is twice continuously differentiable. We show that, depending on the parameters, there are at most two possible payoffs for the seller in a Markov equilibrium, namely, (i) $\underline{b}$ and (ii) the payoff when the seller can commit herself to a single take-it-or-leave-it price. The latter is always an equilibrium payoff; the former requires that $\underline{b}$ be sufficiently large, and,

in particular, does not exist if $\underline{b}$ equals the seller's valuation of zero. We proceed by constructing equilibria which yield these two payoffs and then show that no other payoffs can arise in equilibrium. We then compare the equilibria to the one-buyer case.

A. No-Switching Equilibrium

In this equilibrium, the seller charges $\underline{b}$ each period regardless of the history of the game, and the seller never switches away from the current potential buyer. The buyer of valuation b thus accepts all offers p less than or equal to $(1-\delta_B)b + \delta_B\underline{b}$, so the seller's prior after an offer of p is refused is $\beta(p) = (p-\delta_B\underline{b})/(1-\delta_B)$. A necessary condition for this equilibrium is that the seller does not wish to choose a price above $\underline{b}$, given that $\underline{b}$ is expected next period:

$$\underline{b} > \max_p (p(1-F(\beta(p))) + \delta_S F(\beta(p))\underline{b}). \tag{1}$$

Condition (1) is also sufficient, because if the seller would choose $\underline{b}$ given the current beliefs, she will choose $\underline{b}$ in later subgames in which (by successive skimming) her posterior will be more pessimistic.

B. Switching Equilibrium

In this equilibrium, the seller announces that she will charge a single take-it-or-leave-it price $\hat{p}$ to each buyer, and switch buyers until $\hat{p}$ is accepted. The buyer of valuation b accepts all prices less than or equal to b. The price $\hat{p}$ is chosen to be the "precommitment" price the seller would choose if she could precommit herself to a take-it-or-leave-it price. The fact that $\hat{p}$ is the seller's equilibrium choice without precommitment, but with the buyers playing "truthfully," should be intuitive. Since the seller always switches buyers, her choice problem is stationary, so from dynamic programming we know that the single-period and multi-period optimizations are identical. Formally, let V_S be the seller's value without precommitment,

and let V_S^c be the seller's value with precommitment

$$(2) \qquad V_S = \max_p [(1-F(p))p + F(p)\delta_S V_S]$$

$$V_S^c = \max_p \sum_{t=0}^{\infty} (1-F(p))p\,(F(p)\delta_S)^t = \max_p (1-F(p))p + \delta_S F(p) V_S^c.$$

Since V_S and V_S^c satisfy the same necessary and sufficient functional equation, they have the same unique solution.

Lemma 2: If $F''(p) > 0$, the price which solves (2) is unique.

Proof: $V_S^c = (1-F(p))p/(1-\delta_S F(p))$.

For an interior solution

$$\frac{dV_S^c}{dp} = 0 \Rightarrow (1-\delta_S F(p))\,(1-F(p)-pF'(p)) + (p-F(p)p)\delta_S F'(p) = 0.$$

$$\frac{d^2V_S^c}{dp^2} \propto -(2F'(p) + pF''(p))\,(1-\delta_S F(p)) + (1-F(p) - pF'(p))\delta_S F'(p)$$

$$+ \delta_S F''(p)\,(p-F(p)) + \delta_S F'(p)\,(1-F'(p)p-F(p)) =$$

$$(\delta_S-1)pF''(p) - 2F'(p)\,(1-F(p)\delta_S) < 0$$

at points where $\frac{dV_S^c}{dp} = 0$; i.e., V_S^c is strictly quasi-concave. Q.E.D.

We conclude that the precommitment price(s) are always equilibrium outcomes. Moreover, we can show that if the seller can precommit herself to any time path of prices, her optimal strategy is to charge a fixed price. In outline, the optimal precommitment strategy is to always switch buyers. This has two effects: it maximizes the buyers' reservation values $\beta(p)$ for every price path, and also means every period the seller faces the most favorable distribution of buyer valuations, i.e., $\beta = \bar{b}$. Because the seller always switches, her maximization problem is stationary. Thus a constant price is

optimal. This reproves the Riley-Zeckhauser (1983) result that when a seller can precommit to a price path against an infinite number of potential buyers, the optimal strategy is not to "haggle". We see now that neither precommitment nor Riley-Zeckhauser's assumption of equal search costs (here, discount factors) is necessary; the key is the seller's ability to costlessly switch buyers.

A necessary and sufficient condition for the precommitment price to strictly exceed $\underline{b}$ is

$$\max_p \; p(1-F(p)) + \delta_S F(p)\underline{b} > \underline{b}. \tag{3}$$

This condition overlaps with condition (1), the sufficient condition for a no-switching equilibrium with price $\underline{b}$. The best price may be $\underline{b}$ when buyers expect to face $\underline{b}$ next period; but if buyers expect the seller to switch, their reservation value rises to their valuation, and the return to the seller for charging prices above $\underline{b}$ increases. Thus, for a range of parameter values the "best possible" and "worst possible" outcomes, from the point of view of the seller, can occur. The multiplicity arises from the feedback between the seller's switching decision and buyers' reservation values, and is a general feature of many-player bargaining.

C. Characterization of Equilibria

The "no-switching" and "full switching" equilibrium by no means exhaust the list of possibilities. There can be "semi-switching" equilibria in which the seller randomizes over the switching decision. However, we can show that these two types of equilibria generate all possible Markov equilibrium valuations for the seller; that is, either $V_S = \underline{b}$ or $V_S = V_S^c$.

First we observe that we must have $V_S > \underline{b}$, since the seller can always charge $\underline{b}$ which will be accepted. (This can be proved by a straightforward extension of Lemma 2 of Fudenberg-Levine-Tirole (1985).)

Next, we claim that in any Markov equilibrium, if $V_S > \underline{b}$ then the seller always switches buyers. Were the seller not to switch when price p_t was refused in period t the value of continuing with the "old" seller would have to be at least as great as that of facing a new one. But if p_t is refused with probability strictly between zero and one, the seller is more pessimistic about the old seller than about the new one, and the return to any $p_{t+1} > \underline{b}$ is strictly greater facing the new buyer, because the buyer's reservation value depends only on the current price. Thus either $p_{t+1} = \underline{b} = V_S$, or p_t is refused with probability one so that the seller's beliefs about the old buyer and the new ones are identical. But in a Markov equilibrium there cannot be an offer which is refused with probability one, because then by stationarity the seller would continue to charge the same price each period, and thus would have a payoff of zero. However, since the seller will never charge a negative price, all buyers with valuation less than $p/(1-\delta_B)$ will accept price p, and so the seller's equilibrium payoff must be positive.

We have thus established:

Proposition 1: If delay costs are zero, then either $V_S = \underline{b}$ or $V_S = V_S^c$, the precommitment value. For certain parameter values both are equilibrium payoffs. For $\underline{b}$ sufficiently small, there exists a unique equilibrium. In this equilibrium, the seller never haggles, i.e., she charges a fixed price and switches buyer in case of refusal.

D. Comparison To A Single Buyer

Next we compare the equilibrium outcomes to the case of a single buyer. The condition for an equilibrium with price $\underline{b}$, equation (1), is also necessary and sufficient for an equilibrium with price $\underline{b}$ in the one-buyer case. Moreover, from Fudenberg-Levine-Tirole (1985) we know that with $\underline{b} > 0$ the one-buyer equilibrium is unique. Thus, when (1) is satisfied, the seller is at least as well off facing many buyers. When (1) is not satisfied, the one-buyer equilibrium will yield the seller a payoff greater than $\underline{b}$. However, this value will not be greater than that if the seller could precommit to a price path against the single buyer, which in turn is not greater than the value when the seller can precommit against many buyers. Thus, again the seller is at least as well-off. Conversely, each buyer prefers the seller to have no outside opportunities, because the switching-equilibrium price is never less than that with one buyer. This is very natural — the outside opportunities put the seller in a stronger bargaining position.

More surprising are the efficiency of the switching and the no-switching equilibria for parameter values such that both exist. We show that, depending on the parameter values, either equilibria can yield greater expected total surplus. As all buyers but the first have zero surplus in the no-switching equilibria, this implies that the expected aggregate surplus may decrease in moving from one potential buyer to many of them, despite the fact that the "potential surplus" is larger with many buyers.

To show that more buyers may lower the equilibrium surplus, let $F(b)$ be uniform on $[1,\bar{b}]$, let $\delta_S = \delta_B = \delta$ and take $\bar{b} = 2 - \delta + \varepsilon$. We will show that in this case the precommitment price is just above the no-switching price of 1, that both types of equilibria exist, and that expected total surplus is greater in the no-switching equilibrium.

First, the return to fixed price p, $S(p)$, is given by:

$$(4) \qquad S(p) = \frac{(1-F(p))p}{1-\delta F(p)} = \frac{p(\bar{b}-p)}{(\bar{b}-1)-\delta(p-1)}.$$

Since $F'' = 0$, the seller's maximization problem is strictly quasi-concave, as we remarked earlier.

Now, define ε by the equation $S'(1) = (\bar{b}-1)(\bar{b}-2+\delta) = \varepsilon(\bar{b}-1)$; that is, for $\varepsilon = 0$ the precommitment price is exactly $\underline{b} = 1$. Then since $S(p)$ is concave, we know the optimal precommitment price for $\bar{b} = 2 - \delta + \varepsilon$ is near $\underline{b}$. Furthermore, for these parameter values the no-switching equilibrium exists as well (the right-hand side of (1) is concave, and its derivative is negative at $p=1$).

Last, we compute the derivative of the expected surplus with respect to a fixed price in switching equilibrium (the switching and no-switching surpluses are equal for $p = \underline{b}$). The expected surplus $W(p)$ satisfies

$$(5) \qquad W(p) = \frac{E(b|b\geq p)(1-F(P))}{1-\delta F(p)} = \frac{(\frac{\bar{b}+p}{2})(\bar{b}-p)}{\bar{b}-1-\delta(p-1)}.$$

Thus,

$$(6) \qquad W'(p) \propto -p(\bar{b}-1-\delta(p-1)) + \delta p \frac{(\bar{b}^2-p^2)}{2},$$

and

$$(7) \qquad W'(1) \propto -(\bar{b}-1) + \delta \frac{(\bar{b}^2-1)}{2} =$$

$$(-1 + \frac{\delta(\bar{b}+1)}{2})(\bar{b}-1) =$$

$$(\frac{\bar{b}-1}{2})(\delta(1-\delta+\varepsilon)-2) < 0.$$

Thus, the welfare decreases in moving to the switching equilibrium, or alternatively in moving from one to many buyers. In this example it is easy

to see why surplus decreases with the price: raising the price a bit above $\underline{b}$ does not help the seller very much, because $\underline{b}$ is nearly his best choice, but raising the price does hurt the buyers as a group.

It is much less surprising that expected surplus can be higher with many buyers. The easiest example is when $\delta = 1$, that is, the periods are "short". In this case the seller's return to a fixed price p is p itself (see equation (4)); so the seller charges $\bar{b}$, which equals the expected surplus. With just one buyer the maximum realizable surplus is the buyer's valuation b, and so the maximum expected surplus is the expectation of b.

So we conclude:

Proposition 2: With no delay costs, the seller prefers facing many buyers; the current buyer prefers being the only buyer. Total expected surplus can be bigger or smaller with many buyers than with one.

4. Delay Costs

We now consider the case $d > 0$, so that switching buyers imposes an extra delay cost on the seller. This cost means the seller must become "sufficiently pessimistic" about the current buyer before she finds it worthwhile to switch, so that the "always switch" equilibrium exists for a smaller set of parameter values. The equilibrium path may require the seller to make $n > 1$ offers to the same buyer before switching.

It remains true that if a no-switching equilibrium exists, it is the same as the equilibrium with only one buyer. However, the delay cost makes the seller less eager to switch and thus the no-switching equilibria exists for more values of the parameters. If we let $W_S(b)$ be the seller's valuation in the one-buyer equilibrium when the distribution is truncated to $[\underline{b},b]$ (which exists and is unique from Proposition 1 of Fudenberg-Levine-Tirole (1983)),

then a necessary and sufficient condition for the existence of a no-switching equilibrium is that $\underline{b} > \delta_S^d W_S(\bar{b})$. If this condition is satisfied, then no matter how pessimistic the seller becomes, he will never prefer to switch rather than to charge $\underline{b}$ now. Conversely, if this condition is not satisfied, then after a sufficient number of rejections the seller will strictly prefer to switch (since from Fudenberg-Levine-Tirole we know that in the one-buyer equilibrium the seller eventually charges $\underline{b}$.)

In the no-delay case, the bargaining process has a very special outcome. When $\underline{b}$ is low, the equilibrium is unique with the seller always switching: there is no haggling. When $\underline{b}$ is high, there are other equilibrium in which the seller makes an intermediate offer then either switches or offers $\underline{b}$, for example. This is not haggling in the true sense, because the seller's current valuation is not affected by the buyer's turning down the offer.

Our goal in this section is to show how delay costs can lead to "true" haggling, in which a refusal does lower the seller's current value. We do this by constructing an example. However, we make no effort to classify all equilibria of the game.

In the example, each buyer has one of three values $\underline{b} < \hat{b} < \bar{b}$ with prior probabilities $\underline{\eta}_o, \hat{\eta}_o$ and $\bar{\eta}_o$, which are positive and sum to one. The type of equilibrium we shall consider has an equilibrium path in which the seller first charges a new buyer $p_L > \hat{b}$. If this is rejected he then charges $\hat{b}$. A further rejection results in switching to a new buyer. This type of equilibrium does not exist for all values of the parameters b, δ, d and η. We shall show that this type of equilibrium does exist for a set of parameters with non-empty interior.

Before describing the equilibrium strategies we note some features of the equilibrium path. First, since the seller must have some chance of selling in

the first period, and only $\bar{b}$ will consider an offer $p_L > \hat{b}$, it must be that $\bar{b}$ is willing to pay p_L now rather than $\hat{b}$ next period. Moreover, there is no point in the seller charging a price greater than $\hat{b}$ which $\bar{b}$ would strictly prefer to accept. Therefore, P_L satisfies

$$\bar{b} - p_L = \delta_B(\bar{b}-\hat{b}). \tag{8}$$

Since $\hat{b}$ will be charged next period, there is no reason to charge less. Second, the equilibrium value of the game to the seller is the unique solution V_S of

$$V_S = \bar{\eta}_o p_L + \hat{\eta}_o \delta_S \hat{b} + \underline{\eta}_o \delta_S^{d+2} V_S, \tag{9}$$

since $\bar{b}$ accepts p_L in period one and $\hat{b}$ accepts $\hat{b}$ in period two.

Let p_{min} denote the lowest price ever offered in the past ($p_{min} \equiv \bar{b}$ if the buyer is new), and let p_{t-1} denote the price charged last period. The seller's equilibrium strategy is to switch if $p_{min} < \hat{b}$ the seller switches. If $\hat{b} < p_{min} < p_L$ the seller charges $\hat{b}$ now. If $p_{min} > p_L$, the equilibrium strategy is to randomize between $\hat{b}$ and p_L, depending on the last price offered by the buyer. Note that because the equilibrium strategies depend on the previous period's price, the equilibrium we construct is not Markov; this is frequently the case with a finite number of types (see Fudenberg-Tirole [1983]). Define p_H to be the price that makes the high value buyer $\bar{b}$ indifferent between p_H now and p_L next period.

$$\bar{b} - p_H = \delta_B(\bar{b}-p_L). \tag{10}$$

Note that $p_H < \bar{b}$. If $p_L < p_{min}$ and $p_L < p_{t-1} < p_H$ the seller charges $\hat{b}$ with probability

$$\pi(p_{min}) \equiv (p_H-p_{min})/(p_H-p_L) \tag{11}$$

and p_L with probability $1 - \pi(p_{min})$. As a result if the current price is

the minimum and is in the range $[p_L, p_H]$, $\bar{b}$ is indifferent between buying now and getting the expected price next period. This allows $\bar{b}$ to play a mixed strategy. Finally, if $p_{min} > p_L$ and $p_{t-1} > p_H$, the seller charges p_L.

We turn next to the buyers' strategies. Since the seller never charges less than $\hat{b}$, $\underline{b}$ accepts any offer equal to or below $\underline{b}$ and rejects all higher offers; $\hat{b}$ similarly accepts any offer equal to or below $\hat{b}$ and rejects higher offers. The buyer $\bar{b}$ accepts any offer equal to or below p_L and rejects any offer above p_H. In the intermediate range $(p_L, p_H]$, $\bar{b}$'s behavior depends on the previous history of the game. If a previous offer in this range has been rejected, then $\bar{b}$ rejects this offer. Otherwise $\bar{b}$ randomizes by rejecting with the probability r which makes the seller indifferent between charging $\hat{b}$ and p_L next period. (This in turn enables the seller to randomize as required.) The equilibrium value of r satisfies

$$(12) \qquad (1-r)\bar{\eta}_o p_L + \hat{\eta}_o \delta_S \hat{b} + \underline{\eta}_o \delta_S^{d+2} V_S = [(1-r)\bar{\eta}_o + \hat{\eta}_o]\hat{b} + \underline{\eta}_o \delta_S^{d+1} V_S$$

as we show below (we will choose the parameters such that $r \in [0,1]$).

Given the buyers' strategies, we can now solve for the equilibrium posterior beliefs of the seller. If the current value of p_{min} has been rejected and $p_{min} \leq \hat{b}$, the buyer is $\underline{b}$ so that the posterior is $\underline{\eta} = 1$, $\hat{\eta} = 0$ and $\bar{\eta} = 0$. If the rejected $\hat{b} < p_{min} \leq p_L$ then the buyer cannot be $\bar{b}$ but can be $\hat{b}$ or $\underline{b}$; thus $\underline{\eta} = \underline{\eta}_o/(\underline{\eta}_o + \hat{\eta}_o)$; $\hat{\eta} = \hat{\eta}_o/(\underline{\eta}_o + \hat{\eta}_o)$ and $\bar{\eta} = 0$. If the rejected $p_L < p_{min} \leq p_H$ then $\underline{b}$ and $\hat{b}$ have rejected and $\bar{b}$ rejected with probability r; thus $\underline{\eta} = \underline{\eta}_o/(1-r\bar{\eta}_o)$; $\hat{\eta} = \hat{\eta}_o/(1-r\bar{\eta}_o)$ and $\bar{\eta} = (1-r)\bar{\eta}_o/(1-r\bar{\eta}_o)$. This implies that (12) makes the seller indifferent between selling at p_L and $\hat{b}$ in the period after $\bar{b}$ has been refused.

Finally, to show the suggested strategies are an equilibrium we must show

that $0 < r < 1$ and that the seller's strategy is optimal. This will place restrictions on the parameter values. We will then conclude that there is an open set of parameter values for which these restrictions are satisfied, and thus robust examples of haggling with delay costs.

The first condition on parameters is that $1-r$, computed from (12) for $\bar{\eta}=\bar{\eta}_o$ as

$$1-r = \frac{(1-\delta_S)[\hat{\eta}_o\hat{b} + \underline{\eta}_o\delta_S^{d+1}V_S]}{\bar{\eta}_o(p_L-\hat{b})}, \tag{13}$$

is between zero and one. Next we must check for optimality of the seller's strategy. Given the buyers' strategies, the only prices the seller will consider are $\bar{b}$, $\hat{b}$, p_L and p_H. We must check that

(A) when $p_{min} < \hat{b}$ the seller prefers switching to charging $\bar{b}$. This is equivalent to

$$\underline{b} < \delta_S^{d+1}V_S. \tag{14}$$

(B) When $\hat{b} < p_{min} < p_L$ the seller prefers charging $\hat{b}$ to switching so that

$$\hat{b}\hat{\eta}_o + \underline{\eta}_o\delta_S^{d+1}V_S \geq (\hat{\eta}_o+\underline{\eta}_o)\delta_S^dV_S. \tag{15}$$

(C) When $p_{min} > p_H$ the seller must prefer charging p_L to charging p_H so that

$$V_S \geq r\bar{\eta}_o p_H + ((1-r)\bar{\eta}_o+\hat{\eta}_o)\delta_S\hat{b} + \underline{\eta}_o\delta_S^{d+2}V_S \tag{16}$$

and also to charging $\hat{b}$ so that

$$(1-p)\bar{\eta}_o\hat{b} + \hat{\eta}_o\hat{b} + \underline{\eta}_o\delta_S^{d+1}V_S \leq$$

$$(1-p)\bar{\eta}_o p_L + \hat{\eta}_o\delta_S\hat{b} + \underline{\eta}_o\delta_S^{d+2}V_S$$

for $p = 0$. By inspection we see that this inequality is implied by $p_L > \hat{b}$ and $0 < r < 1$.

(D) When $p_L < p_{min} < p_H$ we already know that the seller is indifferent between charging p_L and $\hat{b}$. We must check the seller does not wish to charge p_H to to switch. Since $\hat{b}$ is better than switching when $\hat{b} < p_{min} < p_L$, it is made even more attractive then $p_{min} > p_L$; esterum (c) p_L is better than p_H when $p_{min} > p_H$; this too is only made more attractive when $p_{min} < p_H$.

Thus, for there to be an equilibrium of the desired type it must be that $0 < 1-r < 1$, (14), (15) and (16) all holding simultaneously. If we can find parameter values b, δ, d and η such that these hold strictly and simultaneously the continuity of (13) to (16) imply they hold for a substantial (open) set of parameters. Our checking can be simplified somewhat if we observe that sufficient condition for (16) (although by no means a necessary condition) is $\delta_B < 1-r$. Thus if we strengthen $0 < 1-r < 1$ to $\delta_B < 1-r < 1$ we may drop (16) and check only this condition, and (14)-(15). If $d = 3$; $\delta_S = \delta_B = 1/2$; $\bar{b} = 5/2$; $\hat{b} = 1$; $\underline{b} = 1/32$ and $\underline{\eta}_o = \hat{\eta}_o = \bar{\eta}_o = 1/3$ then $p_L = 7/4$, $p_H = 17/8$, $1 > V_S > 3/4$ and thus $17/24 > 1-r > 2/3$. Checking (14) we find $\delta_S^{d+1} V_S = 3/64 > \underline{b}$ while (15) may be rewritten as

$$\hat{\eta}_o \hat{b} > \delta_S^d (\hat{\eta}_o + (1-\delta_S)\underline{\eta}_o) V_S \tag{17}$$

for which the left-hand side is 1/3 and the left-hand side no greater than 1/16. This establishes the desired goal: namely the existence of the desired type of equilibrium for an open set of parameters.

So we have obtained robust examples of equilibrium in which the seller haggles for a while, becomes pessimistic and prefers to switch despite the delay cost. This equilibrium closely resembles that derived in the two-period, one-buyer, two-type model of Fudenberg-Tirole [1983] for the case of a tough seller. There, in the notation of this paper, the seller charged p_L in the first period and then $\hat{b}$ in the second. The finite horizon which is

exogenous in Fudenberg-Tirole is derived endogenously here by adding a third type b.

So we obtained equilibria which are intermediate between the no-haggling-constant-switching equilibrium of the zero delay cost case (see section 2) and the haggling-no-switching equilibrium of the traditional infinite delay cost case [see, e.g., Fudenberg-Levine-Tirole (1985)].

Proposition 3: With strictly postitive but finite delay costs, the seller may haggle and switch.

5. Conclusion

We have seen that a seller can credibly play a take-it-or-leave-it strategy when bargaining with many buyers between whom the seller can costlessly switch. Thus, with many "perfectly substitutable" buyers the seller can do as well as if she could precommit herself to a bargaining strategy. If the seller incurs a delay cost in switching negotiations to another buyer, then a take-it-or-leave-it strategy need not be credible, and the seller may make several offers to the same buyer before switching. In either case, the equilibrium need not be efficient. Our result contrasts with those of Shaked-Sutton (1984), who study bargaining with two buyers alternating offers with one seller under complete information. (Actually, they reverse the role of buyers and sellers.) The equilibrium in Shaked-Sutton is efficient, as would be expected with complete information, so that in particular switching never occurs. Our results highlight the way this conclusion changes when information is incomplete.

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